SIMLIFIED CALCULATION OF MEAN QRS VECTOR (MEAN ELECTRICAL AXIS OF HEART) OF ELECTROCARDIOGRAM

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Abstract : In clinical practice assessment of the mean QRS axis (MQRSA) provides information related either with hypertrophy of the ventricles or conduction blocks. The method adopted by clinicians i.e. the inspection of the QRS voltage in six of the limb leads has inherent element of subjectivity of approximately 10°. Moreover, in certain condition, when there is ambiguity about differentiation of left axis deviation assessed by inspection method in to either hypertrophy of left ventricles or complete/hemi block of the left bundle branches, accurate measurement of the axis becomes necessary to arrive at the correct diagnosis. Though a formula based on area under R wave and S-wave of the same QRS complex has been derived for accurate measurement of axis, considering its use in the computer software, working with ordinary electrocardiograph the only method for accurate measurement of the QRS axis is plotting method i. e. the net voltages in Lead-I, and III on their respective axes which is not practicable in clinical settings. Although, calculation of MQRSA by area method gives an accurate assessment of MQRSA, some authors prefer measurement of axis by voltage method, as in cases of the right ventricular hypertrophy with a broad S-wave calculation of axis by area method may give erroneous results. Hence, to obtain correct measurement of MQRSA, we have derived a simplified formula based on the net voltage of QRS complexes in Lead-I and Lead-III. The formula derived is as follows, $Tan\theta = \dfrac{\text{I} + 2III}{\sqrt{3}I}$,

where I and III represent net voltage in Lead-I and III, θ = angle subtended with the axis Lead-I. The value of θ can be found by using scientific calculator or the table. In case net voltage of QRS complex in Lead-I being negative, the value of the θ should be subtracted from 180° to find the angle of mean QRS vector.

Key words : degree of mean QRS axis   mathematical calculation

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INTRODUCTION

Electrocardiography is used to elucidate cardiac arrythmias and conduction defects, and to diagnose and localize myocardial hypertrophy, ischemia and infarction. Assessment of degree of axis of mean QRS vector (mean electrical axis of the heart) is one of the important parameter to be studied in ECG, as it provides information about the conduction defects and also the hypertrophy of the ventricles of heart. Usually the cardiologists and physicians are able to determine this axis just by inspecting the ECG recording in six of the limb leads within a range of 10° of the actual mean QRS axis (MQRSA) without influencing the diagnosis and localization of the defect (1). But in instances of the left axis deviation of about −30°, differentiation between left ventricular enlargement and left incomplete hemi block (2) and of minimal Right axis deviation, accurate assessment of MQRSA is of paramount importance considering the management of the case.

A simple but accurate method for assessment of the MQRSA is plotting the net voltage (voltage of R wave- of the S wave of the same QRS complex) of QRS complex in Lead-I and Lead-III on the axis of the respective leads (3), as described in detail in the section of the "Methods". However, plotting of vectors and then measuring the angle is not practically possible in clinical setting, hence to make the same method. Practicable, we have derived a simple formula which would help in calculating the MQRSA with accuracy in different types of clinical settings.

METHODS

This method is based on the measurement of mean QRS vector using the net voltage (R–S) in Lead-I and Lead-III of the Cartesian co-ordinate system. In Fig. 1 a simple sketch of these leads have been shown. XOX' is as line called X-axis. Take a line OB making an angle of 60° with the X-axis. A line is drawn perpendicular to XOX' from B meeting X-axis in E, where $OE = l_1$ and length BE is denoted by y_1. Produced OB to C in the 3rd quadrant, where $OC = l_3$. Draw a perpendicular CT from C upon X-axis. OT is in –ve direction of X-axis and length TC is in the third quadrant, here both are –ve in sign
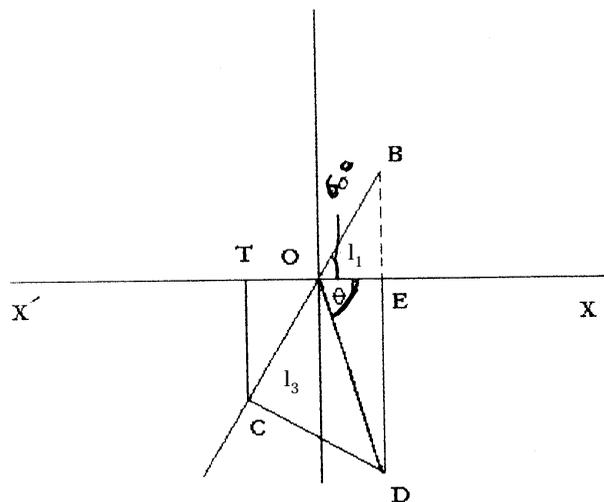

Fig. 1 : Diagrammatic representation of the method adopted in the Cartesian co-ordinate system.

$$\frac{OT}{OC} = CosCOT \Rightarrow \frac{OT}{l_3} = Cos\ 240° \Rightarrow OT = -\frac{l_3}{2} \ ....(1)$$

$$\frac{CT}{OC} = SinCOT \Rightarrow \frac{CT}{l_3} = Sin\ 240° \Rightarrow CT = -\frac{\sqrt{3}l_3}{2} \ ....(2)$$

Thus coordinates of point C are

$$\left(\frac{l_3}{2}, -\frac{\sqrt{3}}{2} l_3 \right) \qquad \text{.................} \quad (3)$$

If θ be the angle a line makes in anticlockwise direction with the X-axis then $m = \tan\theta$ is called the slope of the line. If a line be perpendicular to the above line the slope of the line is $-\dfrac{1}{m}$. Here OB has slope $\tan 60 = \sqrt{3}$, hence line perpendicular to OB will have slope $-\dfrac{1}{\sqrt{3}}$.

Therefore, the line through C perpendicular to COB has slope $= -\dfrac{1}{\sqrt{3}}$.

Equation of line through point (x_1, y_1) is given by $y - y_1 = m(x - x_1)$, where m is its slope.

Therefore, for line CD the equation is

$$y - \left(\frac{\sqrt{3}l_3}{2} \right) = -\frac{1}{\sqrt{3}} \left(x - \left(\frac{l_3}{2} \right) \right)$$

$$\Rightarrow y + \frac{x}{\sqrt{3}} = -\frac{2l_3}{\sqrt{3}} \qquad \text{.................} \quad (4)$$

For the coordinates of point D solve $x = l_1$ (Equation of line BD) with

$$y + \frac{x}{\sqrt{3}} + \frac{2l_3}{\sqrt{3}} = 0 \Rightarrow y + \frac{l_1}{\sqrt{3}} + \frac{2l_3}{\sqrt{3}} = 0 \Rightarrow y - \left(\frac{l_1 + 2l_3}{\sqrt{3}} \right)$$

Therefore, the co-ordinates of point D are $\left(l_1, -\dfrac{l_1 + 2l_3}{\sqrt{3}} \right)$ .... (5)

Equation of line through two points $x_1 y_1$ and $x_2 y_2$ in co-ordinate geometry is

$$\frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2} \qquad \text{.................} \quad (6)$$

Here O has coordinates $(x_1 = 0, \ y_1 = 0)$ and D has coordinates

$$x_2 = l_2, \ y_2 = -\frac{l_1 + 2l_3}{\sqrt{3}}$$

Therefore, the equation of the line OD becomes

$$\Rightarrow y + \frac{l_1 + 2l_3}{\sqrt{3}l_1} x = 0 \qquad \text{.................} \quad (7)$$

Lead-I axis is XOX' which is taken to be X-axis. Lead-III axis is BOC, which is mathematically described by a straight line making an angle of 120° with X-axis in clockwise direction. The net voltage of the QRS vector in Lead-I is shown by distance OE (l_1) on X-axis (Lead-I). The net voltage of QRS vector in Lead-III is shown by the distance OC (l_3) along the straight line BOC. A line perpendicular to XOX' axis passing through E and another line perpendicular to BOC passing through C are obtained using the method co-ordinate geometry in Euclidean space and their intersection point D is obtained. If the co-ordinates of the point D are assumed to be (x_1, y_1) then equation of the line OD (mean QRS vector axis) is

$$\frac{y}{y_1} = \frac{x}{x_1}, \ \text{where } x_1 = l_1 \text{ and } y_1 = -\frac{l_1 + 2l_3}{\sqrt{3}}$$

which gives

$$\sqrt{3}y + \left(1 + \frac{2l_3}{l_1} \right)x = 0 \qquad \text{.................} \quad (8)$$

The angle of QRS axis theta is given by

$$\tan\theta = -\frac{y_1}{x_1} = \frac{l_1 + 2l_3}{\sqrt{3}l_1} \qquad \text{.................} \quad (9)$$

l_1 = net voltage in Lead-I = (I) and l_3 = net voltage in Lead-III = (III) this equation can be rewritten as

$$Tan\theta = \frac{I + 2III}{\sqrt{3}I} \qquad .... (10)$$

The angle theta is measured from the line of QRS vector moving in anti-clock wise direction till we reach the line XOX' (axis of Lead-I). The angle between the QRS vector and Lead-I (XOX') is given by θ.

Using this equation the mean QRS vector can be calculated using the scientific calculator function tan^{-1} (inverse of tan theta).

If l_1 (I) = –ve the angle that comes out from equation 3 is between QRS and OX' while the angle between QRS vector and OX will be supplement of the angle i.e. axis of mean QRS vector will be 180° – θ, as shown in example III in the "Results" section.

RESULTS

The validity of the formula thus derived for calculation of MQRSA can be established by comparing the results of calculation of the axis by this formula and other accepted and established methods meant for the purpose as shown below.

EXAMPLE – I : (Fig. 2, ECG recorded on computerized electrocardiograph) QRS Axis +19° and the same calculated by formula using the voltage in Lead-I (7.5) and Lead-III (–1.5) is 19.10° (Axis within normal limits).

L-I   L-III
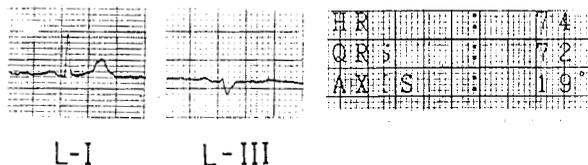
Fig. 2 : A computer based ECG record and analysis of a normal axis (+19°).

EXAMPLE – II : (Fig. 3, ECG recorded on computerized electrocardiograph) QRS Axis –36°, and the same calculated by formula using the voltage in Lead-I (2.2) and Lead-III (–2.5) is –36.3° (Left axis deviation).
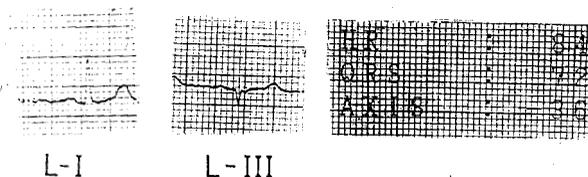

L-I   L-III

Fig. 3 : A computer based ECG recording and analysis of axis indicating left axis deviation (–36°).

EXAMPLE – III : (Fig. 4, shows ECG record in Lead-I and Lead-III taken by an ordinary electrocardiograph) Voltage in Lead-I = –2.5, Lead-III = 2, θ calculated is –19.10° since the voltage in Lead-I is negative the QRS axis would be 180–19.1 = 160.9°. (Right axis deviation)
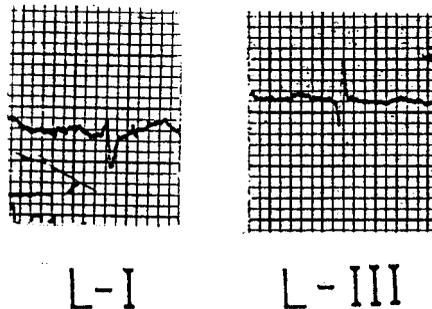

L-I   L-III

Fig. 4 : ECG record showing the Right axis deviation (net voltage in Lead-I is –ve, Axis = +160.9°).

DISCUSSION

It is thus possible to calculate the MQRSA accurately by using the formula derived in the present paper, as the results of this formula are comparable with existing method of accurate measurement by plotting the voltage on axes of Lead-I and Lead-III or calculated by the computer software. Since all calculations involved can be performed on scientific calculator, this method is practically possible in different types of clinical settings using ordinary electrocardiograph, hence this method of MQRSA calculation would be more useful in peripheral hospitals rather than the tertiary level hospitals. Though, a method of mathematical calculation of MQRSA has been suggested based on the calculation of area under R wave and S wave in all the three bipolar limb leads it is not being used by the cardiologists and the physicians, perhaps for the reason of complexity involved in calculation of area under the waves and they still rely upon the inspection method in clinics to get instant assessment of MQRSA but in the process they have to compromise with the accuracy. However, some authors still recommend calculation of the MQRSA by the voltage method, rather than the area method as area method may give an erroneous result in cases of right ventricular hypertrophy with broad S wave (4), therefore calculation of the MQRSA by this formula is more reliable for finding out the MQRSA in cases of right ventricular hypertrophy. The accurate assessment of the MQRSA by this method can also help the physicians, of the primary as well as secondary level hospitals without Echocardiography facility; to assess the response of the anti-hypertensives e.g. angiotensin converting enzyme inhibitors in hypertensive left ventricular hypertrophy, in form of normalization of the MQRSA i.e. from left shift position to verticality of the axis.

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